Common features in phase-space networks of frustrated spin models and lattice-gas models

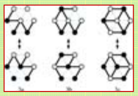
Feng Wang, Yi Peng, and Yilong Han

Department of Physics, The Hong Kong University of Science and Technology

We mapped the phase spaces to complex networks in four models: antiferromagnets on triangular lattices at ground states and above ground states, six-vertex (spin ice) models, 1D and 2D lattice gases. Their phase-space networks share some common features including the Gaussian degree distribution, the Gaussian spectral density, and the small-world properties. The phase spaces exhibit unique self-similar properties. Models with long-range correlations in real space exhibit fractal phase spaces, while models with short-range correlations in real space exhibit nonfractal phase spaces. This behavior agrees with one of the untested assumptions in Tsallis nonextensive statistics even though Tsallis entropy does not apply to these systems. The network community analysis can be used to quantify the weak ergodicity.

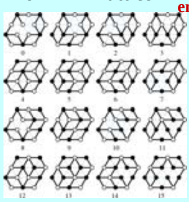
Models:

1. Antiferromagnet on triangular lattice (above the ground state)

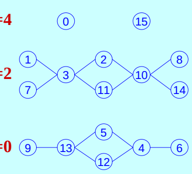


Three types of basic flips, ΔEnergy = 0

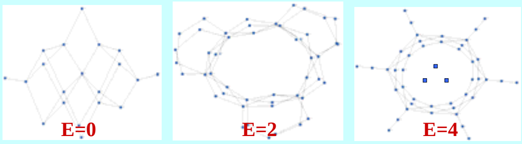
configurations of 2×2×1 lattice



phase space network



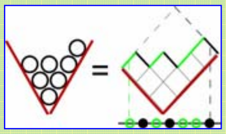
Phase-space networks of 2 × 2 × 2 lattice



2. 1D and 2D lattice gases

2D sphere stacks = 2D square stacks = 1D lattice gas = integer partition

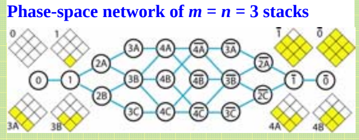
square stacking in an $m \times n$ box = m particles diffuse in $m + n$ sites



$$N_{\text{node}} = \frac{(m+n)!}{m!n!}$$

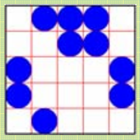
$$N_{\text{edge}} = \frac{(m+n-1)!}{(m-1)!(n-1)!}$$

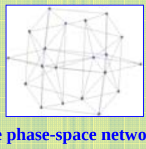
Phase-space network of $m = n = 3$ stacks



2D lattice gas:

- no interaction
- periodic boundary condition





The phase-space network of 3 particles in a 3 × 2 lattice

References:

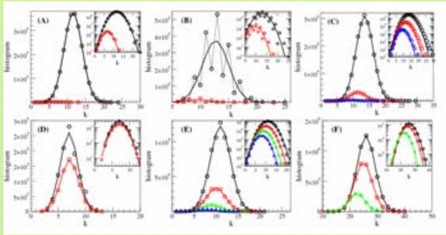
Yilong Han, **Phys. Rev. E** 80, 051102 (**2009**)

Yilong Han, **Phys. Rev. E** 81, 041118 (**2010**)

Yi Peng, Feng Wang, Yilong Han, **Phys. Rev. E** 84, 051105 (2011)

Common features:

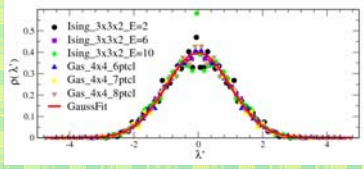
Gaussian degree distribution



The degree, or connectivity, distribution of phase-space networks. (A) cube stacks in boxes with side lengths L = 3, 4. (B) sphere stacks in tetrahedron (L = 5, 6). (C) 4 × 3, 4 × 4, and 4 × 5 spin ices with free boundary conditions. (D) antiferromagnets above ground state in 3 × 3 × 3 and 4 × 3 × 2 lattices. (E) 1D lattice gases ($m = n$ = 8, 9, 10, 11). (F) 2D lattice gases: 8, 10 and 12 particles in 5 × 5 lattices. The Gaussian behavior has been proved in the 1D lattice gas case.

Gaussian spectral density

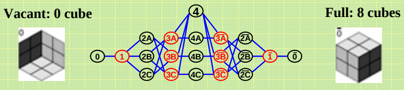
for all phase-space networks



The normalized spectral densities, $\rho(\lambda')$, of phase-space networks of antiferromagets above ground state and 2D lattice gases. The Gaussian $\rho(\lambda')$ reflects the unique topology of phase-space networks.

Small-world property

mean distance between nodes < log (network size N_{node})



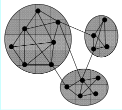
largest distance = number of cubes ~ L^3

$N_{\text{node}} = \exp(N_{\text{spin}} s_0) \sim \exp(L^2 s_0)$ ⟹ small world

s_0: zero point entropy

Community structure

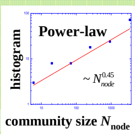
to detect and quantify the weak ergodicity



Dense interconnections within communities

Sparse connections between communities

System trapped in one community for a long time – weak ergodic

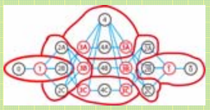


Fractal Property

A basic conjecture in Tsallis statistics:

long-range interacting or correlated systems have fractal phase spaces

Any system with fractal phase space? Here we provide the first examples.



Fractal analysis algorithm: Nature 433, 392 (2005)

1. Generate boxes where all nodes are within a distance l_B
2. Calculate the number of boxes, N_B, needed to cover the network
3. Fractal ⟷ $N_B(l_B) \sim l_B^{-d_B}$, d_B: fractal dimension

Fig. 1 Fractal analysis of phase spaces

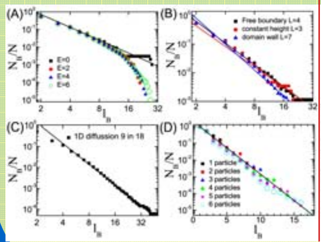
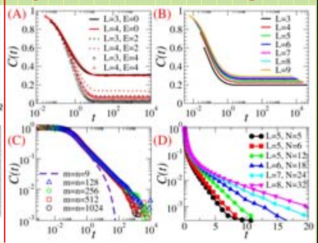


Fig. 2 autocorrelations in real space



	fractal phase space (Fig. 1)	long-range correlation in real space (Fig. 2)
(A) anti-ferromagnets at ground state	yes	yes
(A) anti-ferromagents above ground state (E>0)	no	no
(B) spin ices	yes	yes
(C) 1D lattice gases	yes	yes
(D) 2D lattice gas	no	no